Auction Mechanisms for Traffic Management*

Heiko Schepperle, Christiane Barz, Klemens Böhm, Jonas Kunze, Carolina M. Laborde, Stefan Seifert, Kendra Stockmar

International Graduate School of Information Management and Market Engineering (IME), Universität Karlsruhe (TH)

schepperle@ipd.uka.de, barz@wior.uni-karlsruhe.de, boehm@ipd.uka.de, jonas.kunze@em.uni-karlsruhe.de, carolina.laborde@ira.uka.de, stefan.seifert@iw.uni-karlsruhe.de, kendra.stockmar@ira.uka.de

For many cities, traffic management is nowadays a major challenge. Advances in hardware and software development have resulted in intelligent driver assistance systems which can be used for traffic management. In principal, this allows the design of sophisticated mechanisms which also take the drivers’ valuations of waiting time into account.

In this paper, we investigate a market-based approach to regulate the flow of traffic and estimate the potential efficiency gains of a simple auction mechanism which is designed in a way that minimizes legal obstacles.

1 Scenario

We consider a road traffic intersection with $m$ directions. There is only one incoming and one outgoing lane for each direction. Vehicles arrive from all directions, cross the intersection and leave on an arbitrary outgoing lane.

For simplicity we make the following assumptions: (1) Overtaking is not possible. (2) Only one vehicle can cross the intersection at a time. (3) The time for crossing the intersection is the same for all vehicles and independent of the direction chosen by a vehicle. (4) We do not take into account physical aspects like speed and acceleration.

The exclusive right to cross an intersection for a certain period of time is given to only one vehicle at a time. We call this period *time slot*. This implies that time slots do not overlap and can be numbered in ascending order. Instead of using the period we simply refer to the number of the time slot. Thus, a smaller time slot number indicates an earlier time slot and a higher slot number a later time slot.

Every driver has a valuation of time and, thus, associates opportunity costs with waiting at the intersection. The valuation is the amount of money a driver is willing to pay if he was allowed to cross the intersection one time slot earlier and the valuations

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* This work is partially funded by the German Research Foundation (DFG) and the INIT Innovative Informatikanwendungen in Transport-, Verkehrs- und Leitsystemen GmbH.
differ among drivers. We assume that both the valuations and the interarrival times of vehicles are given by stochastic processes \( \{V_l^i, i \in N\} \) and \( \{T_l^i, i \in N\} \) for all lanes \( l = 1, \ldots, m \).

Since we aim at taking into account the drivers’ valuations of time, we minimize the average waiting time weighted with the respective valuations. Let \( n_l \) be the number of vehicles arriving at lane \( l \) and denote the waiting time and the valuation of the \( i \)th vehicle arriving at lane \( l \) by \( w_l^i \) and \( v_l^i \), respectively. Then the average weighted waiting time (AWWT) is given by

\[
AWWT = \frac{\sum_{l=1}^{m} \sum_{i=1}^{n_l} w_l^i v_l^i}{\sum_{l=1}^{m} n_l}.
\]

The basic idea of this paper is that vehicles are first assigned the right of way in the order of their appearance at the crossing and that they are then allowed to trade, i.e. exchange these (virtual) rights. If there is no trade, our scenario is similar (but not identical) to a U.S. American four-way-stop. We do not look at the process of negotiation and the trade itself, but assume that all vehicles are equipped with some kind of agent-based driver assistance system to which the driver has delegated the trading according to his preferences. We further assume that the negotiations as well as the trade can be conducted in real-time without interrupting the flow of traffic.

2 Legal Aspects

There are quite a few questions associated with the idea of trading the right of way. As noted above, we ignore the technical issues, however, we seek to address two major legal aspects, namely discrimination and negative external effects. From a legal point of view, any system which discriminates between drivers solely based on their valuations might be problematic. The same holds for a system in which a trade between two parties \( A \) and \( B \) potentially prolongs the waiting time of a third party \( C \) (negative external effect).

Discrimination is not given, if vehicles are assigned a time slot depending on their arrival time or lane. But if the driver’s valuation is taken into account, certain drivers could be discriminated and thereby legal objections could arise. Negative external effects are best explained by an example: Imagine a mechanism in which vehicles are allowed to arbitrarily interchange time slots. If a vehicle exchanges an early slot with a later one, not only itself, but potentially also the vehicle behind him has to wait longer.

3 Mechanisms

A mechanism determines a certain order of vehicles crossing the intersection. This order implies a certain value for the \( AWWT \).

3.1 Benchmark

We compare our mechanism with two benchmark mechanisms, a first in first out (FIFO) mechanism and an upper bound of the theoretical optimum. The FIFO mech-
anism relates to the case that trade does not take place. Note that in this mechanism all vehicles are treated independently of their valuations and trading of time slots is not possible. Thus, there is neither discrimination nor are there negative external effects.

The upper bound of the theoretical optimum is computed by a heuristic approach: if no new vehicles arrive, the problem of minimizing $AWWT$ corresponds to a job-sequencing problem with linear delay penalties and linear precedence constraints. An optimal solution for this case is given in Horn (1972). Since this algorithm takes a particular waiting queue as given and is not based on the ex-ante expected arrivals, Horn’s algorithm provides only an upper bound of the theoretical optimum. Moreover, Horn’s algorithm does not treat drivers equally, i.e. it is discriminative. Therefore, applying an optimal mechanism might result in legal difficulties.

3.2 A Simple Auction Mechanism

A simple auction mechanism (SAM), which adheres to the legal restrictions, can be characterized as follows: upon arrival at the crossing, all vehicles receive a time slot according to the FIFO mechanism. Once a vehicle is to cross the intersection, it can auction its slot by collecting bids from other vehicles. If the vehicle in front accepts a bid, it exchanges its slot with the winning bidder and also receives the offered monetary compensation. Since vehicles can, but need not trade their time slots, this auction mechanism is not discriminative. Rational drivers will only accept a trade if they are better-off than in the FIFO benchmark. Note that the bids are evaluated based on both the time slot offered in exchange as well as the offered price.

In order to avoid negative external effects, we impose the following (rather strong) restriction: Only those vehicles which are first in line may bid in the auction. Further, they are allowed to bid only if they did arrive at the crossing before the vehicle which is waiting behind the one auctioning its right of way.

4 Evaluation

To evaluate the SAM and compare it with the FIFO mechanism and the heuristic, we run a computer simulation for an intersection with $m = 4$ directions and the following further parameters: The time period to cross the intersection is normalized at 1 time unit. For the interarrival times of each lane $l = 1, \ldots, 4$ we use an exponential distribution with mean $\frac{1}{3.8}$. Thus, the total average interarrival time for all four lanes together is $\frac{4}{3.8} = \frac{1}{0.95}$ and thereby higher than the serving time of the intersection. For the valuations of vehicles we also use an exponential distribution with mean 1 (independent of the lane).

In every run, we start with an empty system. After an initial phase of $a = 250$ time units, the system is observed for another 1,500 time units.

Table 1 displays the average values and the standard derivations of the $AWWT$ after $n = 720$ simulation runs. Moreover conservative confidence intervals (using Tscheby-
cheff’s inequality and a confidence level of 95%) are given. The table shows that an optimal mechanisms has the potential to reduce the AWWT by 2.54 time units (24%).

Since common random numbers were used in the simulation, the respective differences to the FIFO mechanism give a more concise picture on the relative performance of the SAM (Table 2). The results indicate that the rather strong restrictions, which were introduced due to legal considerations, significantly limit the potential benefits of a market-based approach for traffic management: our SAM realizes only 22% of the potential efficiency gain (or 5.2% of the AWWT).

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<th>Table 1: mean, standard deviation and confidence interval of AWWT</th>
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<th>Table 2: mean, standard deviation and confidence interval of differences in AWWT compared to FIFO ((\Delta AWWT)).</th>
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<td>FIFO vs. Heuristic</td>
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5 Conclusion

In this paper we investigate a market-based approach for traffic management at intersections which also takes the drivers’ valuations of time into account. The presented simple auction mechanism is designed such that legal problems related to discrimination and negative external effects are avoided. A numeric computer simulation shows that by applying the simple auction mechanism the average weighted waiting time can be reduced. The achieved benefits, due to the fact that our approach is still very restrictive, are rather small compared to the potential efficiency gains of an optimal mechanism. Therefore, we will continue our work by extending our mechanisms.

References